LNCC – National Laboratory for Scientific Computing

A Seasonal Mathematical Model For Malaria Spreading With Partial Health Care

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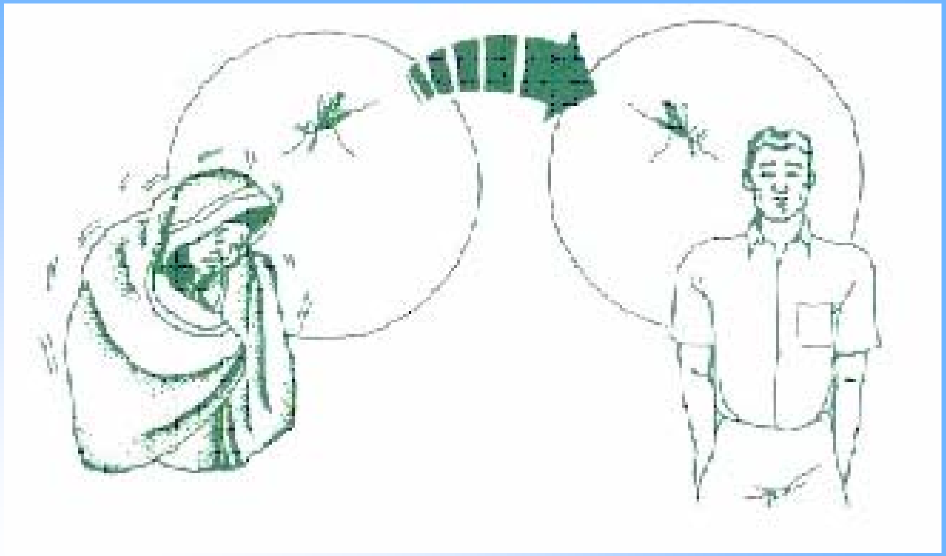
Indirect Transmission



Anopheles darlingi – main vector of Amazonia

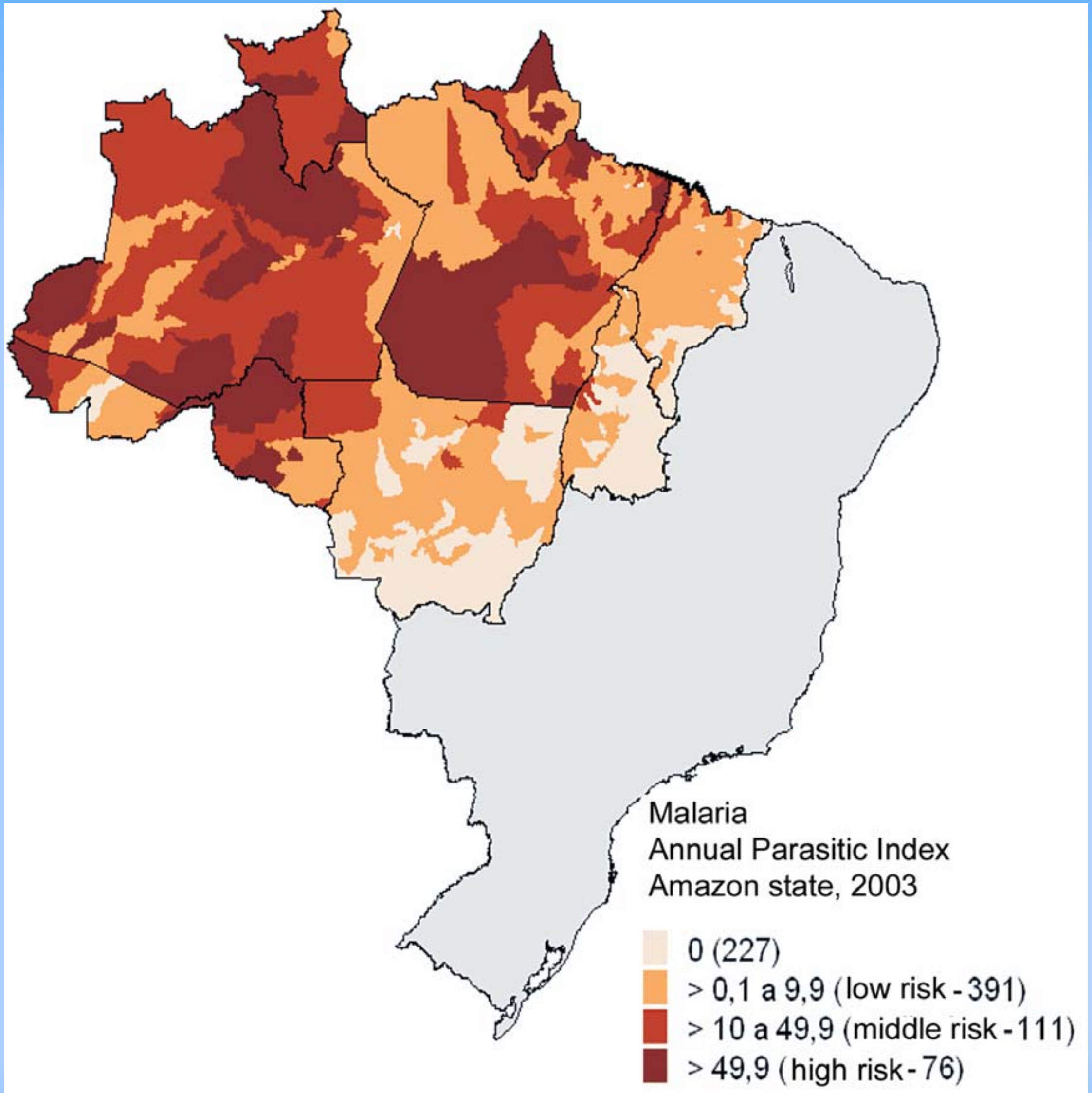


Fig. 1: malaria risk areas in Amazon states, Brazil, 2003.

Source: Sivep-Malária. Ministry of Health.

Mathematical model of mosquito population dynamic

Seasonality in the immature forms and constant density-depending

$$\frac{dV(t)}{dt} = \left(\varepsilon(t) - \delta\right)V(t) - \frac{\left(\varepsilon - \delta\right)}{k}V(t)^2 - \overline{\delta}V(t)$$

where:

$V : \Re^+ \mapsto \Re^+$ is the populational density of mosquito adult female at $t \in \Re^+$;

$\left(\varepsilon(t) - \delta\right) : \Re^+ \mapsto \Re^+$ or $\left(\varepsilon - \delta\right) \in \Re^+$ is the diference between the female recruitment rate to the adult age and her natural death rate;

$k \in \Re^+$ is the carrying capacity;

$\overline{\delta} \in \Re^+$ is the death rate induced by predation .

Equilibrium point ➡ $V^*(t) = k\left(\frac{\varepsilon(t) - \delta - \overline{\delta}}{\varepsilon - \delta}\right)$

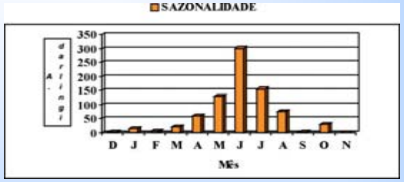
If $\overline{\delta} = 0$, then $V^*(t) = k\left(\frac{\varepsilon(t) - \delta}{\varepsilon - \delta}\right)$. In this case the population is osciling.

Parameters Estimation

- If $t = 0$ (initial month), the rate is minimum: $\dfrac{1}{\varepsilon + \varepsilon_0}$
- If $t = 3$ ou $t = 9$ (third or nineth month), the rate is medium: $\dfrac{1}{\varepsilon}$
- If $t = 6$ (sixth month), the rate is maximum: $\dfrac{1}{\varepsilon - \varepsilon_0}$

$$\varepsilon(t) = \frac{1}{\varepsilon + \varepsilon_0 \cos\left(\frac{\pi}{6} t\right)}$$

The carrying capacity is estimated as the average on the mosquitoes population density in March and September.



$$k = \frac{1}{2}\big(V(3) + V(9)\big) \approx 30$$

Source: PPD, MCT, 1997

Mathematical model of mosquito population dynamic

Parameter estimation

In the absense of any limiting factor,

$$\frac{dV}{dt} = \varepsilon V$$

Solution: $V = V(0)e^{\varepsilon t}$

Then: $\varepsilon = \frac{1}{t}\ln\left(\frac{V}{V(0)}\right)$

In the absense of any growth factor,

$$\frac{dV(t)}{dt} = -(\delta + \bar{\delta})\, V(t)$$

Solution: $V(t) = V(0)e^{-(\delta+\bar{\delta})t}$

Then: $\delta + \bar{\delta} = -\frac{1}{t}\ln\left(\frac{V(t)}{V(0)}\right)$

Biological cicle egg – adult: 0.51/month
Survival rate egg – adult: 57%
Number of eggs/female: 220 eggs (50% males and 50% females)
Survival rate diary of adult mosquitoes: 80.4%

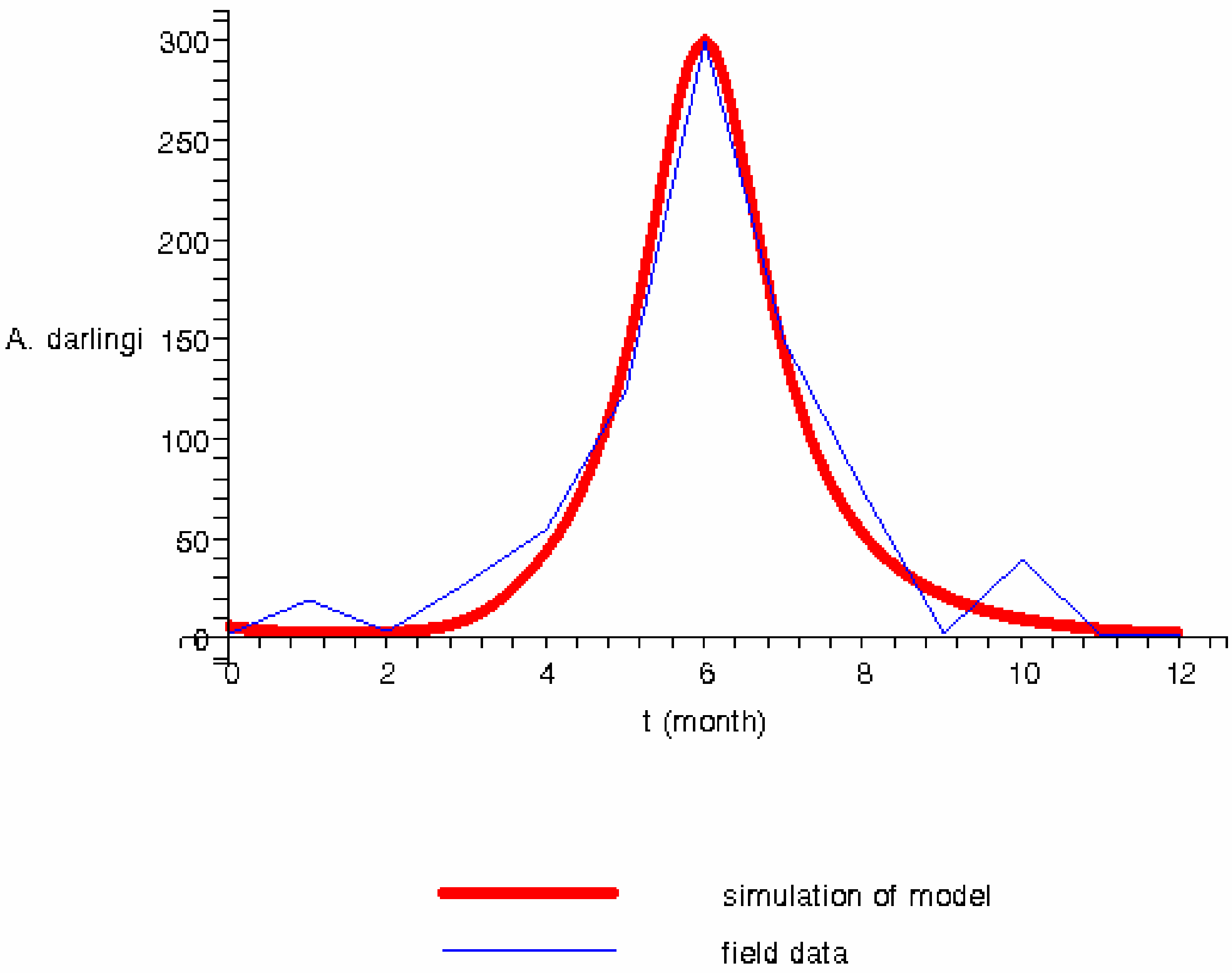
$$\varepsilon = \frac{1}{0{,}51}\ln\left(\frac{220 * 0{,}5 * 0{,}57}{1}\right) = 8{,}114 / mês\,.$$

$$\varepsilon_0 < \varepsilon$$

$$\delta + \bar{\delta} = -\frac{1}{1}\ln\left(0{,}804^{30}\right) = -30\ln(0{,}804) = 6{,}545 / mês\,.$$

$$\delta = 4.545 / month$$

$$\bar{\delta} = 2 / month.$$

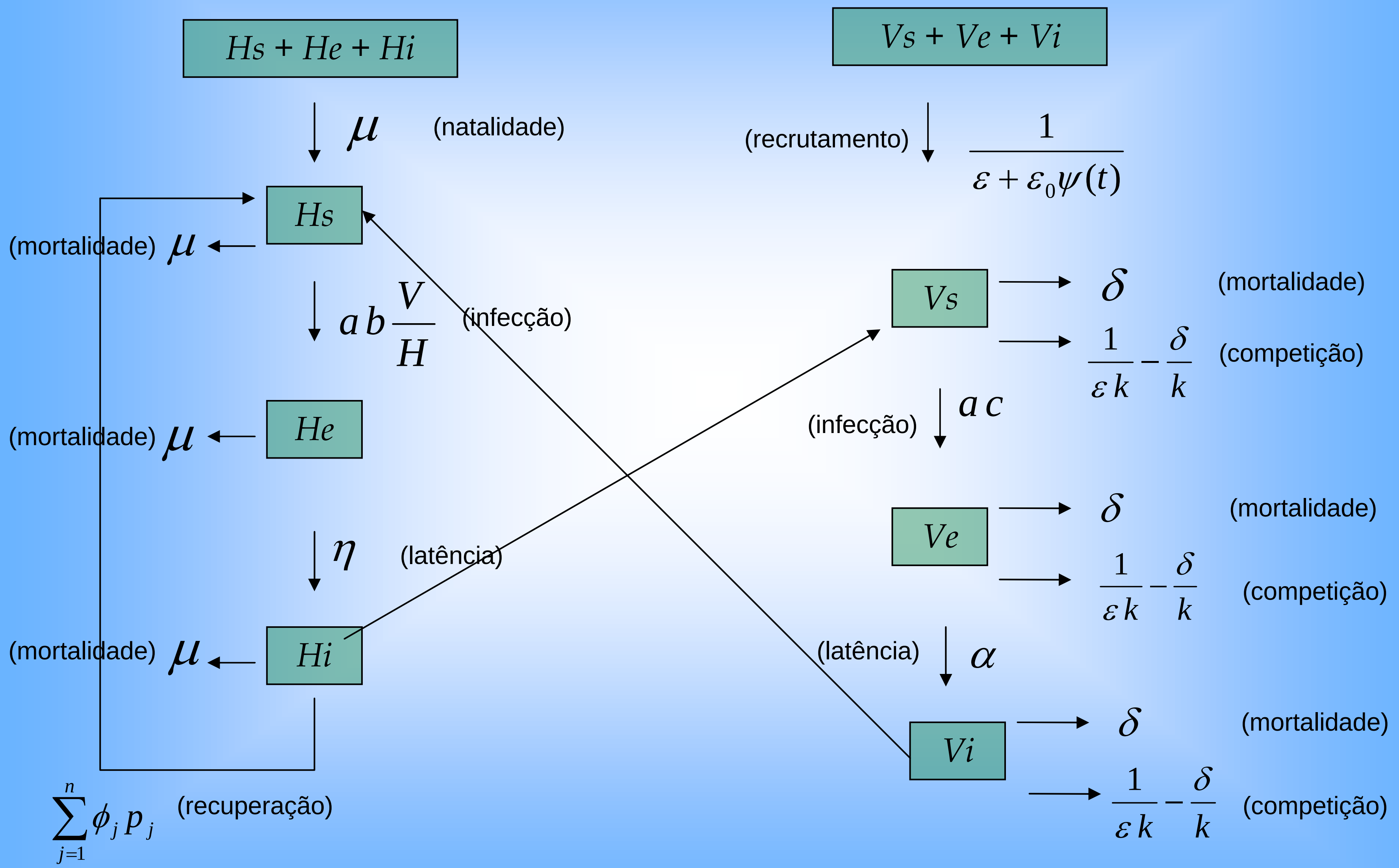


300
250
200
150
100
50
0
A. darlingi
0
2
4
6
8
10
12
t (month)
simulation of model
field data

Hipothesis of model

- Humans – constant size;
- Mosquitoes – varying size;
- Humans – both of sex and ages;
- Mosquitoes – only adult females;
- Without migration in humans or mosquitoes;
- Homogeneous mixing;
- Without immunity;
- The fecundity rate for infected people is the same;
- Whithout mortality in infected people;
- Both of humans and mosquitoes are born susceptible;
- The recovery rate is depending on treatment type;
- The seasonality is considered;

Modelo Matemático



The Mathematical Model

Humans: susceptibles – exposed – infectious

$$
\begin{cases}
\dfrac{dHs}{dt} = \mu(Hs + He + Hi) - abVi\dfrac{Hs}{(Hs + He + Hi)} - \mu Hs + \displaystyle\sum_{j=1}^{n} \phi_j p_j Hi \\[2em]
\dfrac{dHe}{dt} = abVi\dfrac{Hs}{(Hs + He + Hi)} - \mu He - \eta He \\[2em]
\dfrac{dHi}{dt} = \eta He - \mu Hi - \displaystyle\sum_{j=1}^{n} \phi_j p_j Hi
\end{cases}
$$

birth

infection

death

recovery

latency

death

The Mathematical Model

Mosquitoes: susceptibles – exposed – infectious

$$
\begin{cases}
\dfrac{dVs}{dt} = \varepsilon(t)\left(Vs + Ve + Vi\right) - f\left(Vs, Ve, Vi\right)Vs - acVs\dfrac{Hi}{\left(Hs + He + Hi\right)} \\[2ex]
\dfrac{dVe}{dt} = acVs\dfrac{Hi}{\left(Hs + He + Hi\right)} - f\left(Vs, Ve, Vi\right)Ve - \alpha Ve \\[2ex]
\dfrac{dVi}{dt} = \alpha Ve - f\left(Vs, Ve, Vi\right)Vi
\end{cases}
$$

recruitment

mortality

infection

mortality

latency

The Mathematical Model

$$H(t) = Hs(t) + He(t) + Hi(t)$$

$$V(t) = Vs(t) + Ve(t) + Vi(t)$$

$$\begin{cases} \dfrac{dH(t)}{dt} = 0 \\ \dfrac{dV(t)}{dt} = \varepsilon(t)V - f(V)V \end{cases}$$

$$\frac{dV(t)}{dt} = (\varepsilon(t) - \delta)V(t) - \frac{(\varepsilon - \delta)}{k}V(t)^2 - \overline{\delta}V(t)$$

Estimation

Life expectancy of human population: 60 years

Then: $\mu = \frac{1}{720} = 0,00139 / month$

<u>Biting rate</u>

$$a = \frac{hbi}{gc}$$

where hbi is the human bitting index and gc is the gonotrophic cycle.

$b,\ c \in [0,1]$

We take $b = c = 1$

Latent period –10 days for human and 5 (1) days for mosquitoes:

Estimation

Infectious period: period since the end of latent period until the elimination of gametocytes in the organism.

treatment

- efficient

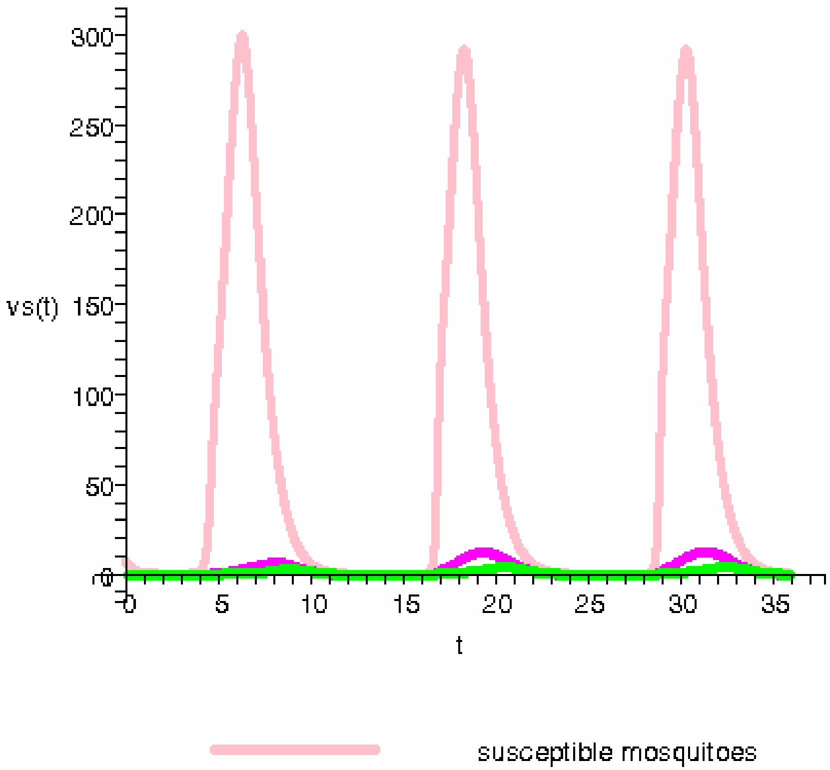
$$\phi_1 = \frac{1}{15+5} * 30 = 1{,}5 / mês$$

- not efficient

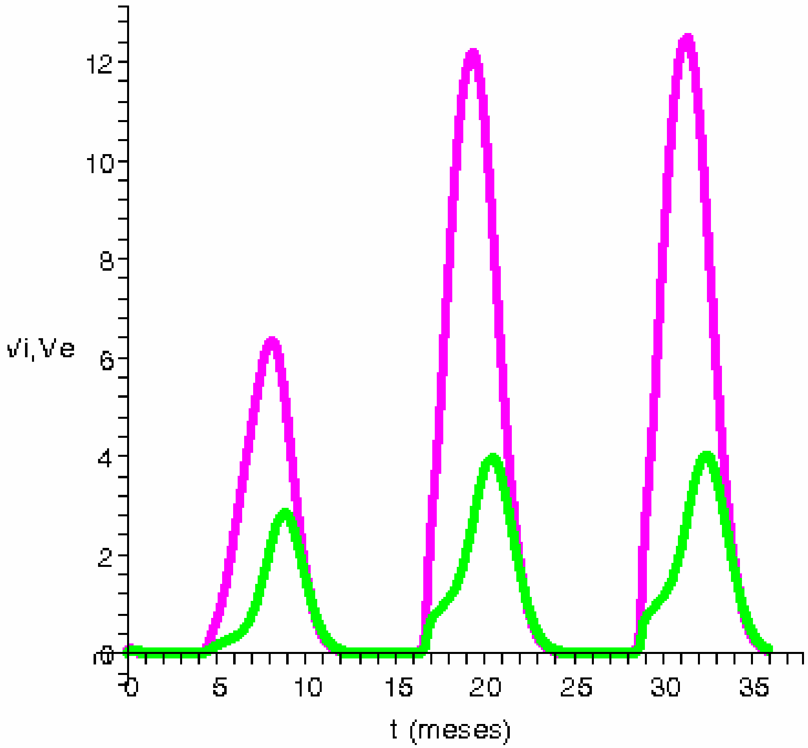
$$\phi_2 = \frac{1}{24} = 0{,}0416 / mês$$

- doesn't exist

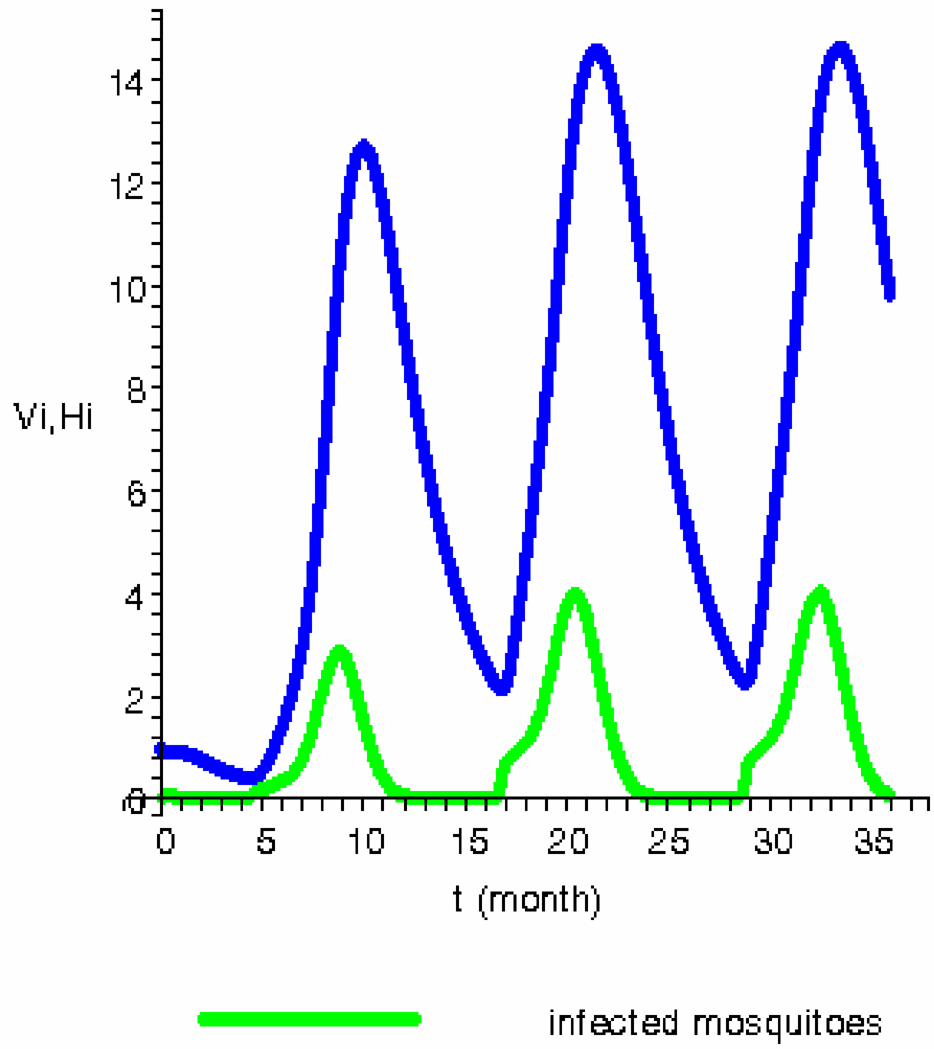
$$\phi_3 = 0$$



300
250
200
150
100
50
0
vs(t)
0
5
10
15
20
25
30
35
t
susceptible mosquitoes

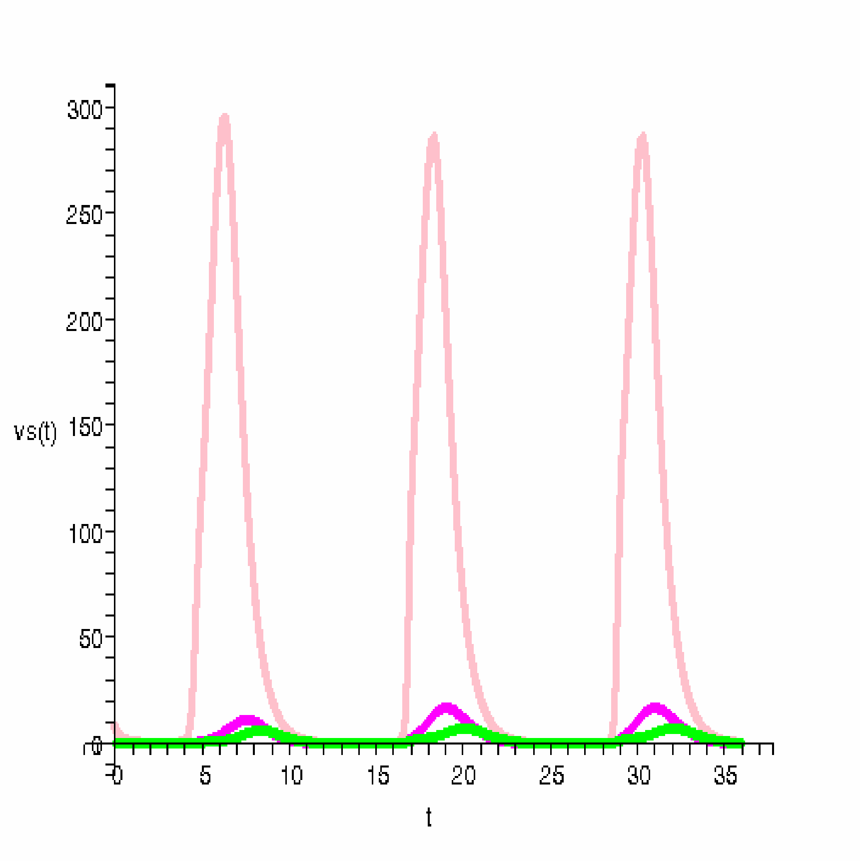


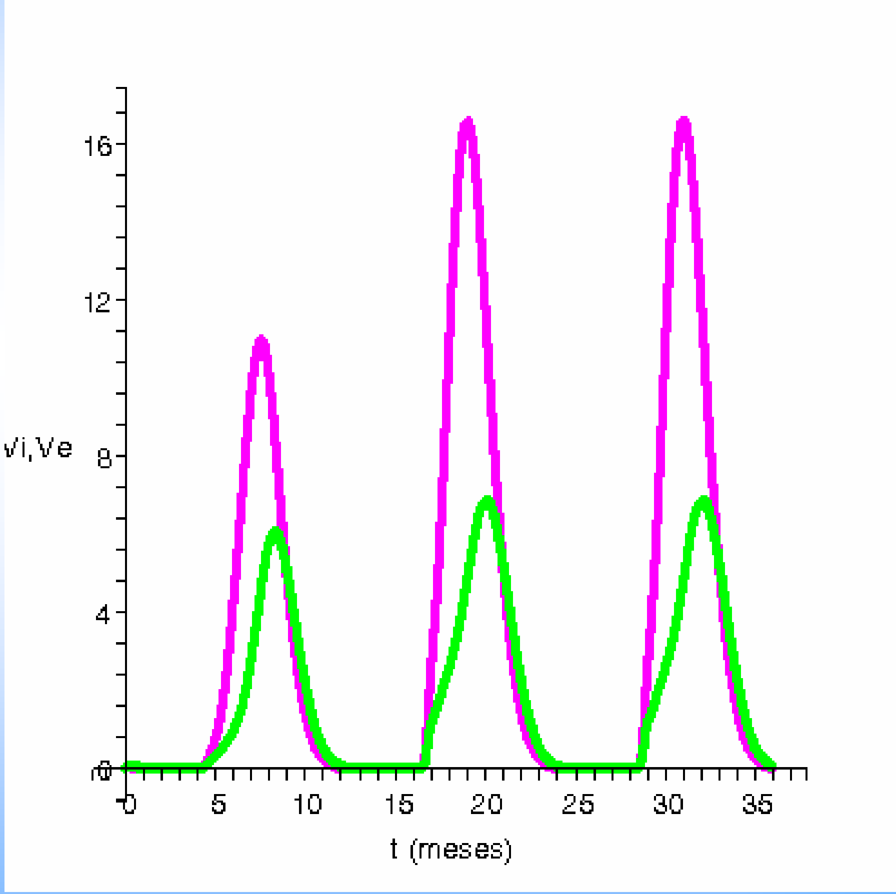
12
10
8
6
4
2
0
Vi,Ve
0
5
10
15
20
25
30
35
t (meses)



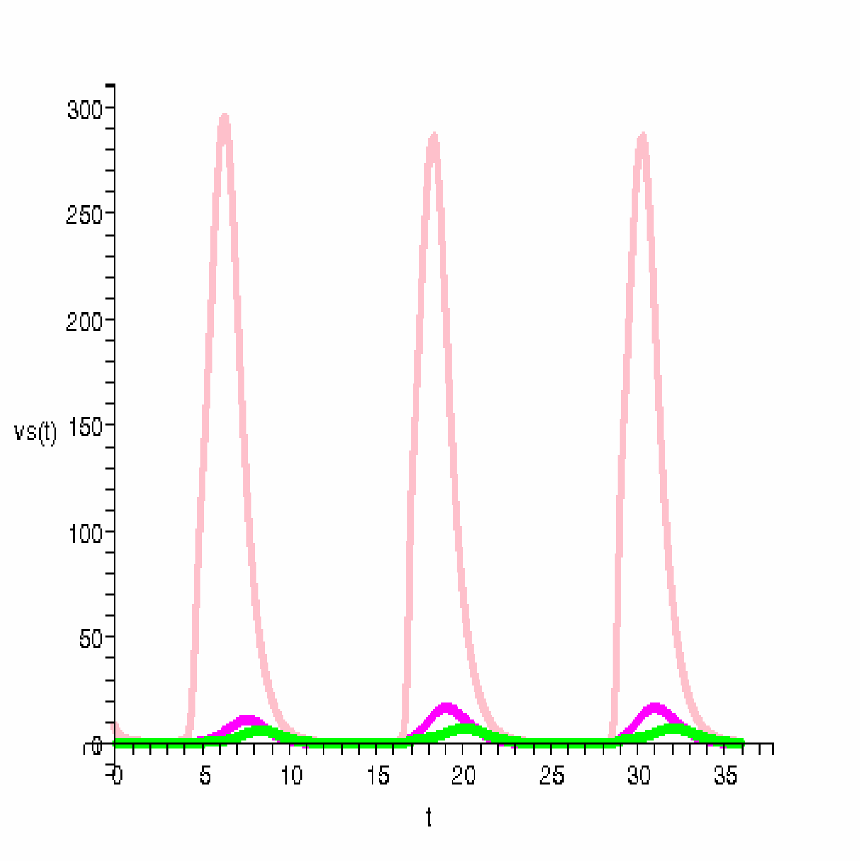
Vi,Hi
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30
35
t (month)
infected mosquitoes

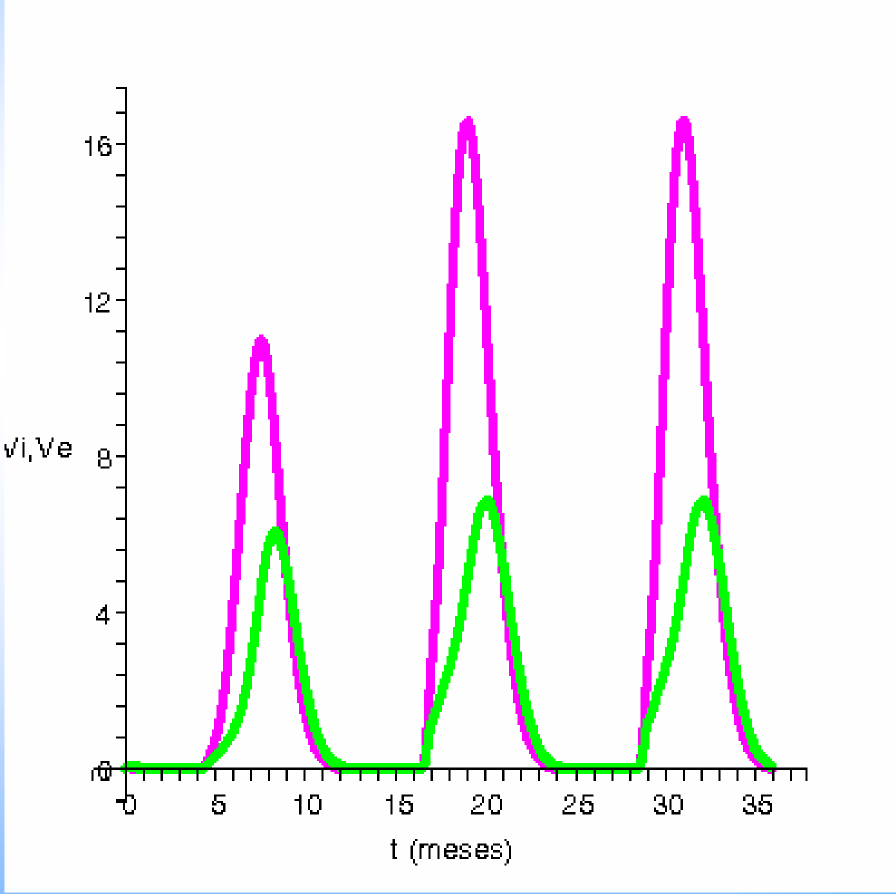
Increasing temperature



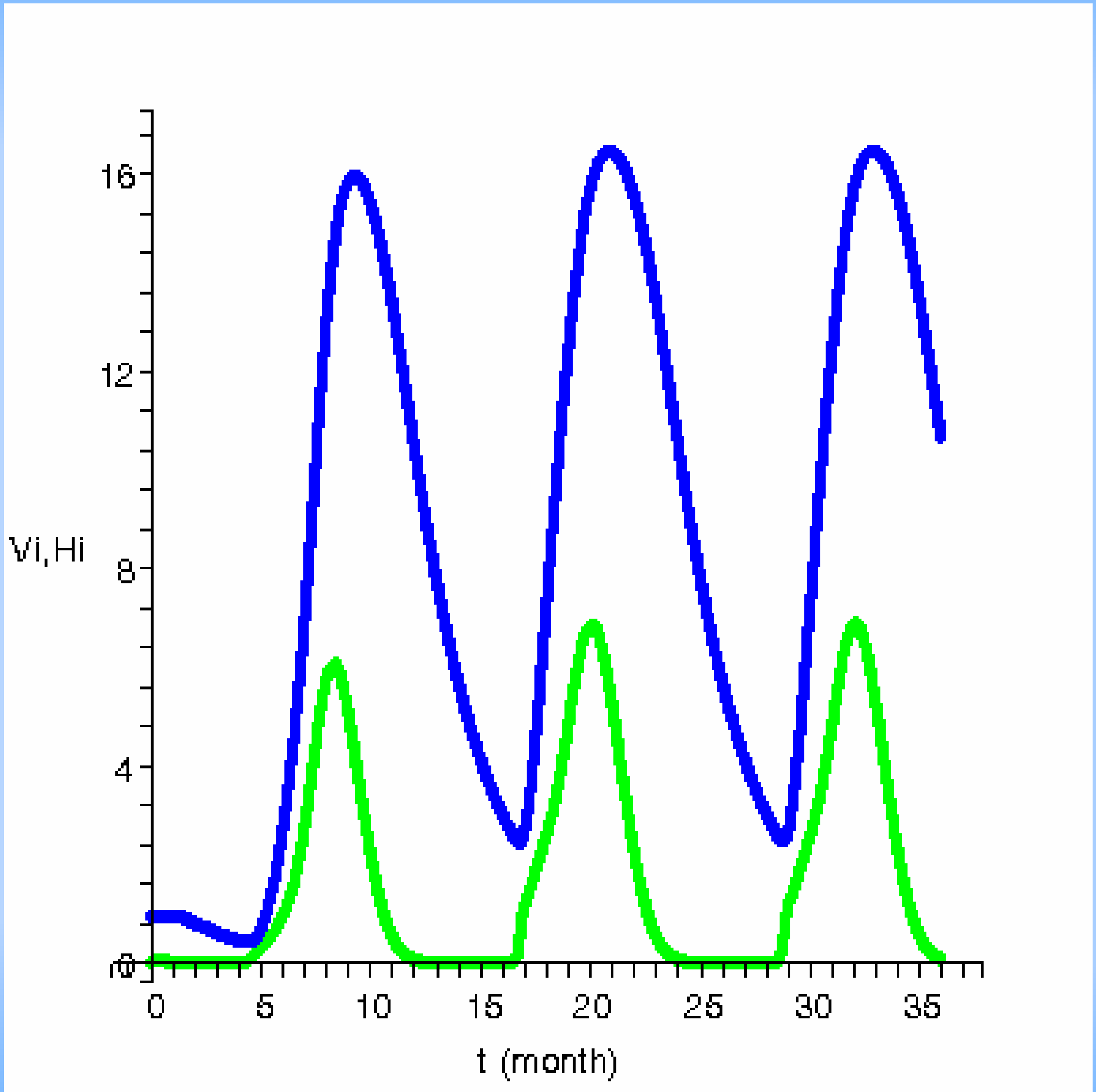


Increasing temperature





Thanks!


Vi,Hi
0
4
8
12
16
0
5
10
15
20
25
30
35
t (month)