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# A new application of Dynamic Data Driven System in the Talbot-Ogden Model for Groundwater Infiltration

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**Abstract**

The Talbot–Ogden model is a mass conservative method to simulate flow of a wetting liquid in variably-saturated porous media. The principal feature of this model is the discretization of the moisture content domain into bins. This paper gives an analysis of the relationship between the number of bins and the computed flux. Under the circumstances of discrete bins and discontinuous wetting fronts, we show that fluxes increase with the number of bins. We then apply this analysis to the continuous case and get an upper bound of the difference of infiltration rates when the number of bins tends to infinity. We also extend this model by creating a two dimensional moisture content domain so that there exists a probability distribution of the moisture content for different soil systems. With these theoretical and experimental results and using a Dynamic Data Driven Application System (DDDAS), sensors can be put in soils to detect the infiltration fluxes, which are important to compute the proper number of bins for a specific soil system and predict fluxes. Using this feedback control loop, the extended Talbot–Ogden model can be made more efficient for estimating infiltration into soils.

*Keywords*: Bin; Data; Flux; Infiltration; Moisture content domain

## 1. Introduction

It is well known that about 99% of domestic water supply in almost every country comes from groundwater [1] [2]. Two active research fields are the study of groundwater recharge and discharge [3] [4], which affect both the quantity and quality of groundwater. These studies are very important in natural environments, city planning, wildlife management, industries, and agriculture. The work in this paper mainly focuses on the recharge process, which is crucial in preventing groundwater from being exhausted. Infiltration is the process by which water on the ground surface enters the soil. The Richards equation [5] is a classical model to describe infiltration, but solving it usually causes mass error and its numerical solution converges only under restrictive conditions [6]. As a replacement for the Richards equation and based on the Green–Ampt model [7], the Talbot–Ogden (T–O) model [8] is a mass-conservative method to calculate the infiltration flux in different unsaturated soil systems. It is also

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inexpensive in terms of computation. The DDDAS [9] approach seeks to optimize the T–O model by incorporating real time sensor data into running simulations.

In the T–O model, the number of bins used to discretize the domain can affect the calculated flux [8]. This model is different from traditional methods because it does not require a discretization of the spatial domain [10] [11], which is a typical technique when solving Richards equation in software packages like Hydrus-1D [12]. The T–O model provides a method that applies to any soil system. Not only is it mass-conservative, but it is also explicit in time. However, for any specific soil, calculation of infiltration fluxes that can approach real field data using measured soil parameters, the correct discretization defined by number of bins required. In this paper, we show how dynamic data can be applied to identify the correct discretization.

The T–O model can also be extended by adding another moisture content axis. This extension expresses porosities in a two dimensional moisture content domain and it keeps the spatial dimension. In this extended model, a bounded subdomain can be chosen to approximate real porosity distribution in any soil, which helps to precisely predict the infiltration flux.

This paper is structured as follows. Section 2 gives a detailed description of infiltration flux in the T–O model, analyses its upper bound, and constructs the extended T–O model. Based on the conclusions obtained in Section 2, we show how to obtain a correct number of bins and a proper two dimensional moisture content subdomain by injecting dynamic data in Section 3. Our conclusions are given in Section 4.

## 2. An Analysis of The Talbot–Ogden Model and Its Extension

### *2.1. Introduction to the Talbot–Ogden model*

There are two kinds of movement in this model: infiltration and redistribution. The discretization of the moisture content domain is shown in Fig. 1. $\theta_r$ and $\theta_i$ are the residual moisture content, and the initial moisture content, respectively. $\theta_e$ is the effective porosity. The interval $[\theta_i, \theta_e]$ that we are interested in is split into $n$ bins indexed by $k$. Each bin has the same width denoted by $\Delta\theta$. $\theta_d$ is the rightmost saturated moisture content at time $t$. For the $k$-th bin, its wetting front depth is denoted by $z_k$. For every time step, the infiltration occurs within each bin, which describes the vertical downward movement of water. For example, for the $k$-th bin in Fig. 1, it is governed by

$$\frac{dz_k}{dt} = \frac{1}{\theta_d - \theta_i}\left(\frac{K(\theta_d)\psi(\theta_d)}{z_k} + K(\theta_d)\right). \tag{1}$$

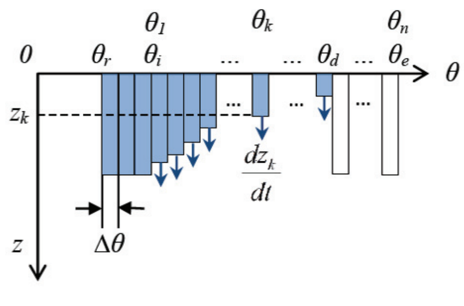



Fig. 1. Infiltration in the Talbot–Ogden model

Immediately after infiltration, redistribution occurs. This process is shown in Fig. 2. The infiltration makes some bins to the right saturated since water tends to infiltrate through large pores. However, the capillary pressure drags water from right bins to ones to the left. Therefore, in Fig. 2, water in the spiking bins is redistributed to the left. But the overall amount of water is not affected by redistribution.

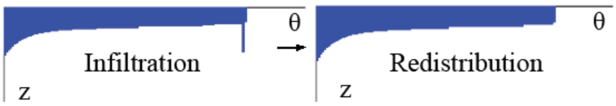



Fig. 2. Infiltration and redistribution within one time step

### *2.2. Infiltration rate analysis*

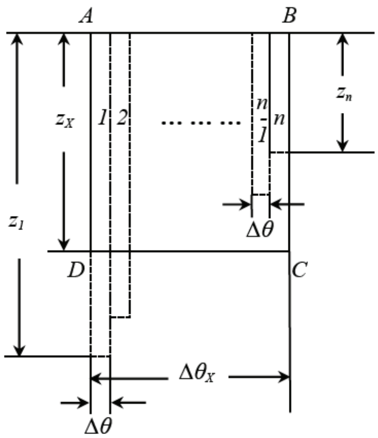



Fig. 3. Discretizing the moisture content domain into *n* bins

In Fig. 3, the rectangle *ABCD* represents one big bin and we denote it by $bin_X$. Its depth and bin width are $z_X$ and $\Delta\theta_X$, respectively. Then $bin_X$ is split into *n* bins, marked by $bin_1$, $bin_2$, …, $bin_{n-1}$ and $bin_n$. All these discretized bins have the same width $\Delta\theta$. Their depths from left to right are $z_1$, $z_2$, …, $z_{n-1}$ and $z_n$. Suppose the water content in the big bin $bin_X$ is equal to that in those *n* bins after discretization, then we have

$$z_X \times \Delta\theta_X = \left(z_1 + z_2 + \cdots + z_n\right) \times \Delta\theta. \quad (2)$$

By Equation (1), the infiltration rate discrepancy between these two cases is

$$\begin{aligned} V_{nbins} - V_{Onebin} &= \sum_{j=1}^{n} V_{bin_j} - V_{bin_X} \\ &= \frac{\Delta\theta \times K(\theta_d) \times \psi(\theta_d)}{\theta_d - \theta_i} \left( \sum_{j=1}^{n} \frac{1}{z_j} - \frac{n}{z_X} \right) \\ &= \frac{\Delta\theta \times K(\theta_d) \times \psi(\theta_d)}{\theta_d - \theta_i} \left( \sum_{j=1}^{n} \frac{1}{z_j} - \frac{n^2}{\sum_{j=1}^{n} z_j} \right), \text{where } z_1 \geq z_2 \geq \cdots \geq z_n > 0 \\ &\geq 0. \end{aligned} \quad (3)$$

From Equation (3), the following is also defined:

$$\left\|V_{nbins}-V_{Onebin}\right\|_{l_1}=V_{nbins}-V_{Onebin}. \tag{4}$$

Therefore, more bins in the model result in a greater infiltration rate.

Now, $n_1$ bins is increased to $n_2$ bins, where $n_2$ and $n_1$ are integers and $n_2 > n_1$. This case resembles the extension from one bin to $[n_2/n_1]$ bins. The difference in infiltration rates between these $n_1$ and $n_2$ bins can be approximated by

$$\left\|V_{n_2 bins}-V_{n_1 bins}\right\|_{l_1}\approx\left\|V_{\left\lceil\frac{n_2}{n_1}\right\rceil bins}-V_{Onebin}\right\|_{l_1}. \tag{5}$$

However, the infiltration rate does not grow to infinity with finer and finer discretization. It is bounded as a function of the leftmost bin and the rightmost bin because these two bins correspond to the slowest flux and the quickest one, respectively. If we do integration from Equation (3) under the assumption that the whole wetting front is like a line segment, we have

$$\begin{aligned}\lim_{n\to\infty}\left(V_{nbins}-V_{Onebin}\right)&=K\left(\theta_d\right)\times\psi\left(\theta_d\right)\times\left(\frac{1}{z_1-z_n}\int_{z_n}^{z_1}\frac{1}{z}\,dz-\frac{z_1-z_n}{\int_{z_n}^{z_1}z\,dz}\right)\\&=K\left(\theta_d\right)\times\psi\left(\theta_d\right)\times\left(\frac{1}{z_1-z_n}ln\frac{z_1}{z_n}-\frac{2}{z_1+z_n}\right).\end{aligned} \tag{6}$$

By calculation, the expression obtained by Equation (6) reaches its maximum if $z_1/z_n$ = 8.16 and $z_n$ is fixed. The maximum obtained can generate an upper bound to the limit in Equation (6). Note that this limit approaches zero when either $z_1/z_n \to 1$ or $z_1/z_n \to \infty$ is true.

We conclude that the infiltration rate increases with the increment of the number of bins, but it is bounded if the number of bins tends to infinity. This is important when we injecting dynamic field infiltration data. If the predicted flux is slower than the actual one, we can increase the number bins by a finer discretization to the moisture content domain. Reversely, if the model generates higher infiltration rate than the actual one, we must make the discretization coarser with fewer bins.

### *2.3. The extension of the Talbot-Ogden model*

To more accurately describe the construction of porosity or moisture content in porous media, we need to build a porosity function of two variables. Fig. 4 shows the method to construct this function.

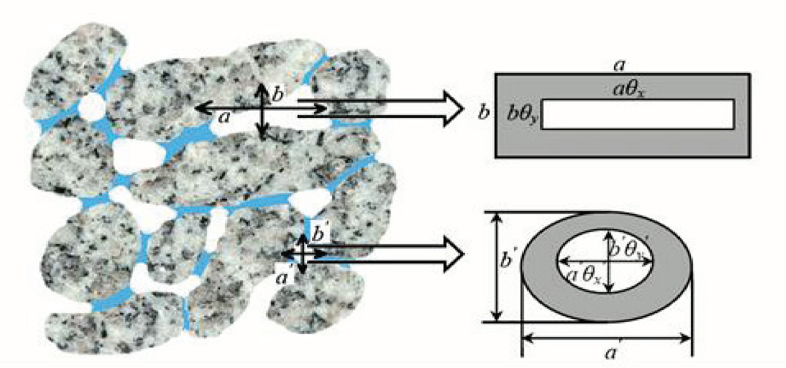



Fig. 4. Measure the porosity or moisture content by two variables

Fig. 4 shows the process of measuring the volume of the void space in a representative volume of soil. First, in the rectangular shaped pore space, the process begins with randomly taking a line segment with length $a$ in the horizontal direction and another one with length $b$ in the vertical direction. The next step is to measure the void space restricted by $a$ in the horizontal direction. Such void space can be occupied by either liquid or air or both. It is denoted by $a\theta_x$, where $\theta_x$ represents the proportion of the horizontal length of the void space to the whole length $a$. Similarly, the other measure $\theta_y$ in the vertical direction can be obtained. Therefore, the moisture content $\theta$ can be rewritten as $\theta = \theta(\theta_x, \theta_y) = \theta_x\theta_y$. In the case of an elliptical-shaped pore space, the process is almost identical, except that $a^{'}$ and $b^{'}$ are used to denote the length of the major and the minor axes, respectively.

By this extension, the infiltration occurs in a three dimensional space consisting of two porosity axes and one spatial dimension. The bins now become vertical long slim cubes. The infiltration in the original T–O model is calculated using Equation (1). The redistribution is directional in this extended model. Fig. 5 shows the directional redistribution.

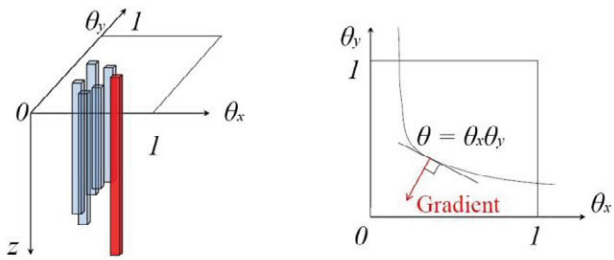



Fig. 5. Directional movement of water in redistribution

Algorithms based on these concepts were created and numerical experiments are undertaken to implement infiltration based on this extended model. Note that in Fig. 5, the moisture content domain is [0, 1] × [0, 1], but it is not necessary to consider the entire unit square. We can take any subdomain covering the interval $[\theta_i, \theta_e]$ within it to simulate the infiltration. Fig. 6 shows a rectangular shaped subdomain ABCD within the unit square. Infiltration happens in the green area bounded by $\theta_i = \theta_x\theta_y$, $\theta_e = \theta_x\theta_y$, and ABCD.

## 3. The Application of Dynamic Data Driven System in Groundwater Movement

DDDAS [9] is a paradigm in which dynamic data are used for improving a running simulation. Conversely, the simulation can be steered to a correct direction. Using DDDAS requires real-time data acquisition and control. If this data driven process is properly done, it increases the predicative capability of the simulation system. For instance, DDDAS is used to optimize laser treatment of prostate cancer [13], and to forecast weather [14]. Therefore, DDDAS is often used in the context of uncertainties [9].

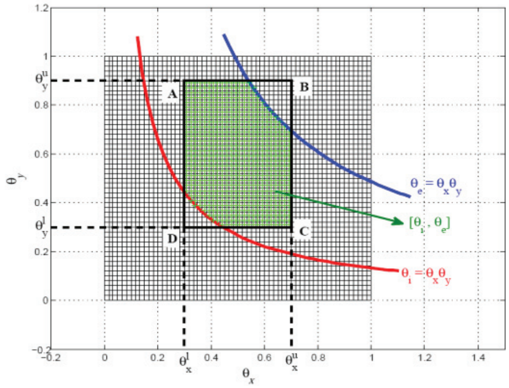



Fig. 6. Directional movement of water in redistribution

If dynamic infiltration data are injected into the running T–O model, different numbers of bins are obtained corresponding to different infiltration rates. Fig. 7 shows the numerical experiments for three different soils with 50, 125, and 250 bins, respectively.

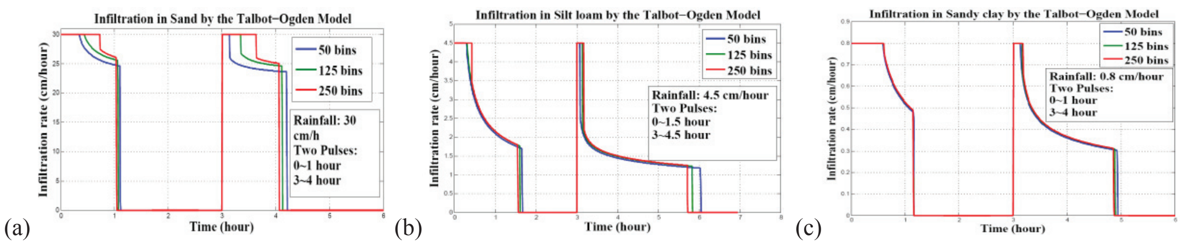



Fig. 7. Infiltration with different numbers of bins in different soils: (a) Sand, (b) Silt loam, and (c) Sandy clay.

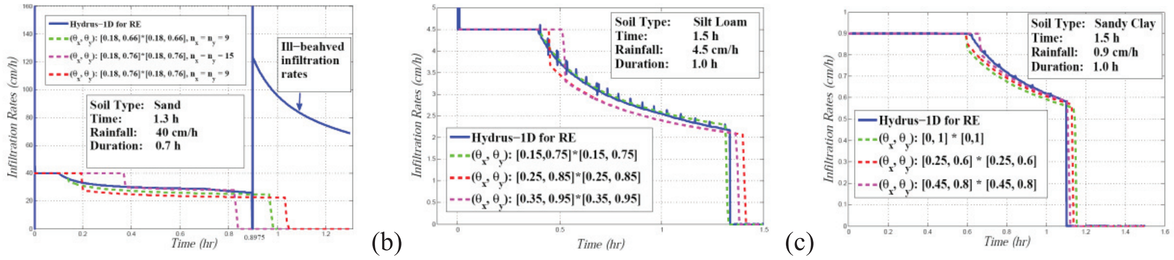
In each picture of Fig. 7, there are two pulses of rainfall occurring and they last for the same period. Water on the surface of ground infiltrates in different soils. The data-driven simulation can be implemented as follows.

- Suppose we can detect real-time data of infiltration rates by sensors in the soil, then we initiate the T–O model at the beginning of the first rainfall with a certain number of bins, i.e. 50 bins in sand.
- In the case of 50 bins, if the water does not start to accumulate on the surface of sand at the time 0.35 h, which we define as ponding time, we need to increase the number of bins in the running simulation, i.e. to 125 bins.
- This modification to the number of bins highly depends on measuring the difference between real-time data and the current running model. If they are matching well, we keep the current discretization. Otherwise, we need to either increase or decrease the number of bins.

When new infiltration flux data are injected into our running simulation, the number of bins should be optimized because Fig. 7 shows the overall infiltration rates increase with the increment of the number of bins for any soil in the T–O model. This feature is more obvious and discernible for coarser soil particles like sand. After several loops of this feedback-control process, a proper number of bins can be obtained so that we can use this identified model to predict future fluxes in soils.

The extended model describes the infiltration under different porosity distributions. The DDDAS can help to identify a proper porosity/moisture content or particle distribution of soil with dynamic infiltration data. This is also like solving an inverse problem. First, we choose a rectangular subdomain as shown in Fig. 6. Second, we discretize this rectangular shaped area to construct vertical bins as shown in Fig. 5. Note that this discretization identifies the number of bins and we make this value unchanged. It is possible to choose a proper number of bins for different soils by referring to [8]. We then start simulating infiltration. Meanwhile, a field experiment may begin. When the flux data from sensor are compared with the model data in the simulation, updates can be made to the subdomain.

We can move the rectangular subdomain in the unit square, enlarge or shrink it to a bigger or smaller size. Fig. 8 shows the simulation of infiltration for three different soils under different subdomains.




(a) (b) (c)

Fig. 8. Comparison of infiltration rates computed by Hydrus-1D and the extended model with different porosity distributions: (a) Sand, (b) Silt loam, and (c) Sandy clay.

The numerical experiments in Fig. 8 show the infiltration rates are larger if the percentages of small porosities are higher in all these three soil textures. However, the higher percentage of big porosities corresponds to a longer ponding time. These features can be used in updating the size and the location of the subdomain after comparing the new field data of infiltration rates with the data obtained from the running model. Fig. 8 (a) also shows the ill-behaved results obtained by Hydrus-1D, which is not mass conservative. With DDDAS, once the subdomain in our extended T–O model is identified and fixed in the moisture content domain, this model can be used to estimate fluxes in soils. Fig. 9 shows one feedback control loop driven by dynamic field data. Fig. 9 (a) describes the application of DDDAS in the original T–O model, while (b) shows its application in the extended model. Nevertheless, to update the number of bins and the infiltration subdomain simultaneously is challenging in the extended model. The reason is because we need to first fix either the number of bins or the rectangle-shaped subdomain, and then alternate the other parameter.

## 4. Discussion and Future Work Plan

In this paper, Section 2 gives a detailed description of infiltration flux in the T–O model, analyses its upper bound, and constructs the extension of the T–O model in the moisture content domain. We conclude that finer discretization will lead to higher infiltration rates, but with bounded increment. We have also shown that the extension in the moisture content domain makes a probability porosity distribution possible. Based on the conclusions obtained in Section 2, we have shown how to obtain a correct number of bins and a proper two dimensional moisture content subdomain by DDDAS in Section 3. The DDDAS can help to generate the proper number of bins or infiltration subdomain for any soil. Therefore, it can improve predicting the infiltration and estimating the porosity/particle distribution in soils.


(a) (b)

Fig. 9. Application of DDDAS in the infiltration simulation: (a) Updating the number of bins in the T–O model (b) Updating the moisture content subdomain in the extended model.

In the future, it is reasonable to obtain time adaptive numbers of bins for our simulation. Since using a fixed number of bins may not be realistic, we must consider different numbers of bins at various moments. For instance, we can combine several bins into one bin or split one bin into several bins whenever necessary. It is also possible to employ this idea to the extended model. Therefore, both a correct number of bins and a proper infiltration subdomain need to be considered simultaneously. To optimize the computation by general purpose graphics processing units is another work remains to be done because updating the running model can be computationally expensive.

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