Colorado State University

# Amorphous Polyhedral Model for Stochastic Control of Autonomous UAVs

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(AFOSR DDDAS Program Review, 2013)

# Outline

- Project timeline: 2013—2016
- Motivation, Objectives & Approach
- Progress & recent results
- Future work & plans
- Conclusions & discussion

# DDDAS + polyhedral + control

Combine two strengths

- Mathematical Models and Algorithms for Stochastic Control of UAVs
- System Software with the Polyhedral Model and extensions

# UAV Control: the Problem

# Challenges

- Dynamic constraints
- Uncertainty and stochasticity
- Spatially varying measurement errors
- Data fusion and geometric synergy
- External factors
  - Obstacles (may also act as occlusions)
  - Wind
  - Aggression & evasion

# Non-myopic Dynamic Control

- Problem is inherently dynamic
  - Must exploit feedback
  - Poor control actions at one time will lead to regret in the future
- non-myopic: cannot just apply control action that optimizes performance at that time instant
- Aligned with DDAS goals

# Solution methodology

- Partially Observable Markov Decision Processes (POMDP)

- Solved using approximation method called NBO (nominal belief-state optimization)

# System Software

- Polyhedral Model
  - Mathematical theory of programs and program transformation
  - Long history
  - Nonlinear optimization for choosing compiler transformations
  - Important in multi- and many-core era
  - Equations are executable programs (declarative language, Alpha)

# History (~30 years)

- Systolic Array Synthesis (mid 80s)
  - Scheduling with affine dependences
  - Localization
  - Closure
- Feautrier 93: seminal papers on
  - Dataflow analysis
  - Scheduling (multidimensional time)
- Code generation
  - Scanning polyhedra
  - Scanning unions of polyhedra
- 2000's: Engineering the polyhedral model
  - Tools, design space exploration
  - (Parametric) tiled code generation
- Workshop on polyhedral compilation techniques
- School http://labexcompilation.ens-lyon.fr/polyhedral-school

# System Software Challenges

- Limited applicability (subset of *static control* programs with *affine* dependences)
  - Iterative algorithms with dynamically determined termination & control
  - Irregular data access patterns (indirection, sparse matrix computations, etc.)
- Dynamic changes to target machines:
  - Faults (increasingly important for exascale)
  - Dynamically changing loads (for multi- and many-cores)

# Proposed Solution

- Extend the expressivity (Alphabets: an extension of Alpha)
  - Iterative termination through unbounded polyhedra & fixed-point semantics
  - Non-affine dependences through uninterpreted functions
  - Rework mathematical closure properties
- Polyhedral compilation to dynamic targets
  - Parametric code generation with late binding of parameters

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# Compiling Alphabets

Alpha(bets) is equational/declarative (single assignment). Compiler analyses:

- Scheduling
- Lifetime (memory allocation)

Static scheduling is undecidable [SQ'89]
fallback strategy: demand-driven evaluation

- Alphabets: unbounded computations ➔ (potentially) unbounded memory
  - For "well formed" programs, the history to be maintained can be statically bounded

# Polyhedral X10 programs

- Explicitly parallel polyhedral programs
- Polyhedral subset of X10 (DARPA HPCS project)
- Challenge: execution order is a partial (not total) order
  - Defined a new "happens before" relation & used that to provide data-race detection algorithm [PPoPP 2013]
  - For X10 programs with synchronizations (clocks) problem becomes undecidable [submitted]

Collaboration with Saraswat (IBM) & Feautrier (ENS Lyon)

# Changing target architectures

"JIT" compilation of polyhedral programs:

Maximize static compilation, leave as many "tunable" parameters.

- Challenge: hierarchical & parametric tiling: (polyhedral model meets its Waterloo)

  - CART: constant aspect ratio tiling

    [in preparation]

# Recent Results

- POMDP algorithmic kernels in Alphabets
  - Particle swarm optimization (PSO)
  - Nelder Mead
- Data association
  - Pre data-association gating
  - Multiple Hypothesis Tracking [in preparation]
- Target tracking with obstacles
  - Changing terrain under the UAV
- Modeling aggressive & evasive targets
  - Target motion law → highly state dependent

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# Future Plans

- Compilation
  - Code generation for dynamic dependences
  - Speculative code generation
  - CART – code generation and design space exploration
    - Optimal run-time tile-size selection
- Stochastic control
  - Address target ambiguity
    - Incorporate the cost of data association
  - Integrate dynamic decision horizons for NBO
- Close the loop: use stochastic control on parallel code
  - to reason about parallel code generated from the polyhedral model
  - dynamically adapt /tune the code

# Conclusion

- Polyhedral model for affine program analysis – we go beyond that to
  - Dynamic dependences
  - Non-polyhedral iteration spaces
- Optimal control using POMDPs provides a foundation for UAV control
  - Mathematically elegant and rigorous
  - Extensions to target ambiguity