

**Amorphous Polyhedral Model
for Stochastic Control of
Autonomous UAVs**

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(AFOSR DDDAS Program Review, 2013)

Outline

- Project timeline: 2013—2016
- Motivation, Objectives & Approach
- Progress & recent results
- Future work & plans
- Conclusions & discussion

DDDAS + polyhedral + control

Combine two strengths

- Mathematical Models and Algorithms for **Stochastic Control of UAVs**
- System Software with the **Polyhedral Model** and extensions

UAV Control: the Problem



Challenges

- Dynamic constraints
- Uncertainty and stochasticity
- Spatially varying measurement errors
- Data fusion and geometric synergy
- External factors
 - Obstacles (may also act as occlusions)
 - Wind
 - Aggression & evasion

Non-myopic Dynamic Control

- Problem is inherently dynamic
 - Must exploit feedback
 - Poor control actions at one time will lead to regret in the future
- non-myopic: cannot just apply control action that optimizes performance at that time instant
- Aligned with DDDAS goals

Solution methodology

- Partially Observable Markov Decision Processes (POMDP)
- Solved using approximation method called NBO (nominal belief-state optimization)

System Software

- Polyhedral Model
 - Mathematical theory of programs and program transformation
 - Long history
 - Nonlinear optimization for choosing compiler transformations
 - Important in multi- and many-core era
 - Equations are executable programs (declarative language, Alpha)

History (~30 years)

- Systolic Array Synthesis (mid 80s)
 - Scheduling with affine dependences
 - Localization
 - Closure
- Feautrier 93: **seminal papers** on
 - Dataflow analysis
 - Scheduling (multidimensional time)
- Code generation
 - Scanning polyhedra
 - Scanning unions of polyhedra
- 2000's: Engineering the polyhedral model
 - Tools, design space exploration
 - (Parametric) tiled code generation
- Workshop on polyhedral compilation techniques
- School <http://labexcompilation.ens-lyon.fr/polyhedral-school>

System Software Challenges

- Limited applicability (subset of *static control* programs with *affine* dependences)
 - Iterative algorithms with dynamically determined termination & control
 - Irregular data access patterns (indirection, sparse matrix computations, etc.)
- Dynamic changes to target machines:
 - Faults (increasingly important for exascale)
 - Dynamically changing loads (for multi- and many-cores)

Proposed Solution

- Extend the expressivity (Alphabets: an extension of Alpha)
 - Iterative termination through **unbounded polyhedra** & **fixed-point semantics**
 - Non-affine dependences through uninterpreted functions
 - Rework **mathematical closure** properties
- Polyhedral compilation to dynamic targets
 - Parametric code generation with **late binding** of parameters

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Compiling Alphabets

Alpha(bets) is equational/declarative (single assignment). Compiler analyses:

- Scheduling
- Lifetime (memory allocation)

Static scheduling is undecidable [SQ'89]

fallback strategy: demand-driven evaluation

- **Alphabets**: unbounded computations →
(potentially) unbounded memory
 - For “well formed” programs, the history to be maintained can be statically bounded

Polyhedral X10 programs

- Explicitly parallel polyhedral programs
- Polyhedral subset of X10 (DARPA HPCS project)
- Challenge: execution order is a partial (not total) order
 - Defined a new “happens before” relation & used that to provide data-race detection algorithm [PPoPP 2013]
 - For X10 programs with synchronizations (clocks) problem becomes undecidable [submitted]

Collaboration with Saraswat (IBM) & Feautrier (ENS Lyon)

Changing target architectures

“JIT” compilation of polyhedral programs:
Maximize static compilation, leave as many
“tunable” parameters.

- Challenge: hierarchical & parametric tiling:
(polyhedral model **meets its Waterloo**)

- **CART: constant aspect ratio tiling**

[in preparation]

Recent Results

- POMDP algorithmic kernels in Alphabets
 - Particle swarm optimization (PSO)
 - Nelder Mead
- Data association
 - Pre data-association gating
 - Multiple Hypothesis Tracking [in preparation]
- Target tracking with obstacles
 - Changing terrain under the UAV
- Modeling aggressive & evasive targets
 - Target motion law → highly state dependent

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Future Plans

- **Compilation**
 - Code generation for dynamic dependences
 - Speculative code generation
 - CART – code generation and design space exploration
 - Optimal run-time tile-size selection
- **Stochastic control**
 - Address target ambiguity
 - Incorporate the cost of data association
 - Integrate dynamic decision horizons for NBO
- **Close the loop: use stochastic control on parallel code**
 - to reason about parallel code generated from the polyhedral model
 - dynamically adapt /tune the code

Conclusion

- Polyhedral model for affine program analysis – we go beyond that to
 - Dynamic dependences
 - Non-polyhedral iteration spaces
- Optimal control using POMDPs provides a foundation for UAV control
 - Mathematically elegant and rigorous
 - Extensions to target ambiguity